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A SIMPLE METHOD OF CONSTRUCTION OF PARTIALLY VARIANCE BALANCED BLOCK DESIGNS

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SUMMARY

A simple procedure of construction of partially variance balanced block designs with even number of treatments is given. The method is illustrated by examples and the incidence matrices of binary and ternary variance (or partially variance) balanced block designs for 4, 6, 8 and 10 treatments are also given along with their Efficiency Factors. The construction of Partially Variance Balanced Binary Block Design (3) with unequal block sizes and unequal replications is also discussed.

Keywords : Incidence Matrix; Variance Balanced; Binary, Ternary, Partially variance Balanced Concurrences; Efficiency factor.

Introduction

Tocher [8] implicitly introduced the idea of variance balanced design and constructed certain ternary balanced designs with equal block sizes. It should be noticed that the Balanced Incomplete Block Design (BIBD) which are binary proper and equireplicate block designs are a special case of variance balanced designs.

Later, Rao [5] fully exploited the concept of variance balanced designs. According to him "A design with v treatments is called Variance Balanced (*VB*) if the design permits the estimation of every elementary contrast of treatments with equal variances". The necessary and sufficient condition for a connected design to be variance balanced is that all the off diagonal elements of the matrix C are equal. Such designs are constructed by Murty and Das [3], Dey [1], Saha and Dey [6], Nigam [4], Saha [7] and Tyagi and Rizwi [9] by using different methods.

The concept of variance balanced design was further generalized to Partially Variance Balanced Design by Tocher [8] when he constructed partially Balanced Ternary Designs with two classes of variances. Mehta, Agarwal and Nigam [2] originated the idea of partially variance balanced n-ary block designs and constructed these with the help of PBIB designs.

"A block design with v treatments and b blocks is called partially variance balanced n-ary block design (m) if

(i) the incidence matrix $N_{v \times b}$ has n distinct entries $f_0, f_1, f_2, \ldots, f_1(n-1)$,
(ii) the sum of ith row is r_i, $(i = 1, 2, \ldots, v)$,
(iii) the sum of jth column is k_j, $(j = 1, 2, \ldots, b)$, and
(iv) the variances of $v(v-1)/2$ elementary constrasts of treatments can be grouped into m different classes. Based on this classification any two treatments are either 1st or 2nd or ... mth associates; the relationship of associates being symmetrical."

In the present work a simple procedure has been developed which leads to the construction of binary and Ternary (variance balanced or partially variance balanced) block designs for even number of treatments. The construction of Partially Variance Balanced Binary Block Design (3) with unequal block sizes and unequal replications is also given.

The design constructed here are more economical in comparison to the existing designs and attains almost the same efficiency factor (EF). It is easy to construct and analyse these designs. They require smaller number of experimental units. For instance, Mehta *et al.* [2] has developed ternary $PV(B)B(2)$ design for 10 treatments in 10 blocks of 8 plots each where as we have obtained a ternary $(PV(B)B(2))$ design for $t = 5$ (10 treatments and 10 blocks) with a block size 7. Saha [7] constructed the design with parameters $v = 8$, $b = 16$, $r = 12$, $k = 6$, where as our design has got the parameters $v = 8$, $b = 8$, $r = k = 6$.

2. Method of Construction

Let the number of treatments be $v = 2t$, where t is a positive integer. we consider a $t \times t$ matrix E with each element equal to unity and a unit matrix I_t, of order t. The incidence matrix of the design can be obtained as below:

$$N = \begin{bmatrix} E_{t \times t} & \vdots & (n-1) I_t \\ \cdots & \cdots & \cdots \\ (n-1) I_t & \vdots & E_{t \times t} \end{bmatrix}$$

This design is binary for $n = 2$ and ternary for $n \geqslant 3$. These binary and ternary designs so constructed will be variance balanced when $(n-1) = \frac{1}{2}t$ with parameters ($b = v = 2t$, $r = k = \frac{3}{2}t$, $\lambda = t$); otherwise they will be partially variance balanced block designs with two classes of variances and will be denoted as $PV(B)BD(2)$. This type of partially variance balanced design has got the following parameters:

$$v = b = 2t, \quad r = k = t + (n-1), \quad \text{when } (n-1) \neq \tfrac{1}{2}t$$

Sum of the concurrences in all the blocks
$$\begin{cases} \lambda_{ll'} = t & \text{where } l' \neq l = 1, 2, \ldots, t \\ \lambda_{uu'} = t & \text{where } u' \neq u = t+1, \ldots, 2t \\ \lambda_{lu} = 2(n-1) & \text{where } l \neq u. \end{cases}$$

3. Examples

The incidence matrices of binary and ternary designs alongwith their efficiency factors corresponding to $t = 2, 3, 4$ and 5 are reproduced below, where rows stand for treatments and columns for blocks:

Binary Design — *Ternary Design*

(i) for $t = 2$,

Binary Design, $EF = 0.889$; Ternary Design, $EF = 0.818$

1	1	1	0	1	1	2	0
1	1	0	1	1	1	0	2
1	0	1	1	2	0	1	1
0	1	1	1	0	2	1	1

(ii) for $t = 3$,

Binary Design, $EF = 0.893$; Ternary Design, $EF = 0.862$

1	1	1	1	0	0	1	1	1	2	0	0
1	1	1	0	1	0	1	1	1	0	2	0
1	1	1	0	0	1	1	1	1	0	0	2
1	0	0	1	1	1	2	0	0	1	1	1
0	1	0	1	1	1	0	2	0	1	1	1
0	0	1	1	1	1	0	0	2	1	1	1

(iii) for $t = 4$,

$EF = 0.896$

1	1	1	1	1	0	0	0
1	1	1	1	0	1	0	0
1	1	1	1	0	0	1	0
1	1	1	1	0	0	0	1
1	0	0	0	1	1	1	1
0	1	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	0	0	1	1	1	1	1

$EF = 0.889$

1	1	1	1	2	0	0	0
1	1	1	1	0	2	0	0
1	1	1	1	0	0	2	0
1	1	1	1	0	0	0	2
2	0	0	0	1	1	1	1
0	2	0	0	1	1	1	1
0	0	2	0	1	1	1	1
0	0	0	2	1	1	1	1

(iv) for $t = 5$

$EF = 0.897$

1	1	1	1	1	1	0	0	0	0
1	1	1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	1	0	0
1	1	1	1	1	0	0	0	1	0
1	1	1	1	1	0	0	0	0	1
1	0	0	0	0	1	1	1	1	1
0	1	0	0	0	1	1	1	1	1
0	0	1	0	0	1	1	1	1	1
0	0	0	1	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1

$EF = 0.906$

1	1	1	1	1	2	0	0	0	0
1	1	1	1	1	0	2	0	0	0
1	1	1	1	1	0	0	2	0	0
1	1	1	1	1	0	0	0	2	0
1	1	1	1	1	0	0	0	0	2
2	0	0	0	0	1	1	1	1	1
0	2	0	0	0	1	1	1	1	1
0	0	2	0	0	1	1	1	1	1
0	0	0	2	0	1	1	1	1	1
0	0	0	0	2	1	1	1	1	1

It is noted that the binary design for $t = 2$ and the ternary design for $t = 4$ are variance balanced and others are $PV(B)BD(2)$ in the examples given above.

4. Concluding Remark

The method of construction of Partially Variance Balanced n-ary Block designs in blocks of different sizes with different replications is reported elsewhere and has been accepted for publication. However, the method of construction of Partially Variance Balanced Binary Block Design (3) with unequal block sizes and unequal replications is given below:

Let N_1, N_2 be the incidence matrices of two BIB designs with parameters $(v_1, b_1, r_1, k_1, \lambda_1)$ and $(v_2, b_2, r_2, k_2, \lambda_2)$ respectively. Then the incidence matrix N^* defined as

$$N^* = \begin{bmatrix} N_{1_{v_1 \times b_1}} & \vdots & E_{v_1 \times b_2} \\ \cdots & \cdots & \cdots \\ E_{v_2 \times b_1} & \vdots & N_{2_{v_2 \times b_2}} \end{bmatrix}$$

will be the incidence matrix of a partially variance balanced binary block design (3) with parameters:

$$v^* = v_1 + v_2, \quad b^* = b_1 + b_2$$

Size of the first b_1 blocks $(1, 2, \ldots, b_1)$, $k^* = k_1 + v_2$ Size of the last b_2 blocks $(b_1 + 1, b_1 + 2, \ldots, b_1 + b_2)$, $k_2^* = k_2 + v_1$ replications of the first v_1 treatments $r_1^* = r_1 + b_2$ replications of the last v_2 treatments $r_2^* = r_2 + b_1$.

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